Hybrid iterative/direct strategies for solving the three-dimensional time-harmonic Maxwell equations discretized by discontinuous Galerkin methods

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This work aims at developing high-performance numerical strategies for the computer simulation of time-harmonic electromagnetic wave propagation problems in complex domains and heterogeneous media. In this context, we are naturally led to consider volumic discretization methods (i.e. finite difference, finite volume or finite element methods) as opposed to surfacic discretization methods (i.e. boundary element method). Most of the related existing works deal with the second-order form of the time-harmonic Maxwell equations discretized by the edge finite element method [15] and more recently, discontinuous Galerkin methods [11]. Recently, theoretical results concerning discontinuous Galerkin methods applied to the time-harmonic Maxwell equations have been obtained by several authors. Most of these results use a mixed formulation [16, 12] but the convergence of discontinuous Galerkin methods on the non-mixed formulation have also been proved [11, 4]. Here, we are concerned with the application of such discontinuous Galerkin methods to the first order form of the three-dimensional time-harmonic Maxwell’s equations, and we aim to design a parallel solution strategy for the resulting large, sparse algebraic systems with complex coefficients.

Indeed, as far as non-trivial propagation problems are considered, classical iterative methods behave very poorly or even fail to converge. The preconditioning issues for highly indefinite and non-symmetric matrices is for instance discussed by Benzi et al. in [3] in the context of incomplete factorization and sparse approximate inverse preconditioners. If a robust and efficient solver is sought then a sparse direct method is the most practical choice. Over the last decade, a significant progress has been made in developing parallel direct methods for solving sparse linear systems, due in particular to advances made in both the combinatorial analysis of Gaussian elimination process, and on the design of parallel block solvers optimized for high-performance computers [2, 10]. However, direct methods still fail to solve very large three-dimensional problems, due to the potentially huge memory requirements for these cases. Iterative methods can be used to overcome this memory problem but, in order to build robust preconditioners, some approaches combine the direct solver techniques with iterative preconditioning techniques. For example, a popular approach in the domain decomposition framework is to use a direct solver inside each subdomain and to use an iterative solver at the interfaces between subdomains. This approach is adopted in this work.

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Domain Decomposition methods are flexible and powerful techniques for the parallel numerical solution of systems of PDEs. Concerning their application to time-harmonic wave propagation problems, the simplest algorithm was proposed by Després [6] for solving the Helmholtz equation and then extended and generalized for the time-harmonic Maxwell equations in [7, 5, 1]. The analysis of a larger class of Schwarz algorithms has been performed recently in [8]. Our ultimate objective is the design and application of optimized Schwarz algorithms in conjunction with discontinuous Galerkin methods. The first step in this direction is to understand and analyze classical overlapping and non-overlapping Schwarz algorithms in the discrete framework of these discretization methods. To our knowledge, except Helluy [9], where such an algorithm is applied to a discretization of the first-order time-harmonic Maxwell equations by a upwind finite volume method, no other attempts for higher order discontinuous Galerkin methods or different kind of fluxes can be found in the literature. A classical domain decomposition strategy is adopted in this study which takes the form of a Schwarz-type algorithm where Després conditions [7] are imposed at the interfaces between neighboring subdomains. A multifrontal sparse direct solver is used at the subdomain level. Furthermore, in order to reduce the memory requirements for storing the L and U factors associated to the factorization of subdomain problems, a mixed-precision approach is adopted where the factors are computed and stored in single precision (32 bits) arithmetic. Then, to recover double precision arithmetic (64 bits), these factors are used either as a preconditioner to a Krylov subspace method or within an iterative refinement procedure. Similar strategies have recently been considered in the linear algebra community essentially for performance issues [13, 14] on modern high-performance processors.

The resulting domain decomposition strategies can be viewed as hybrid iterative/direct solution methods for the large, sparse and complex coefficients algebraic system resulting from the discretization of the time-harmonic Maxwell equations by a discontinuous Galerkin method. We investigate in details the numerical and parallel performances of these strategies by considering on one hand, classical diffraction problems by perfectly electric conductive objects and, on the other hand, the propagation of a plane wave in a heterogeneous medium defined as a realistic geometrical model of the head of a mobile phone user.

References


